Calculus 1
Midterm Exam
October 2, 2023 (18:30 – 20:30)

university of groningen

Please read the instructions!

1) Prove using the ε-δ definition that $\lim_{x\to 4} \dfrac{x^2-2x-8}{x-4} = 6$.

2) Apply l'Hospital's Rule to evaluate the following limit: $\lim_{x\to 0}[\cos(x)]^{\frac{1}{x^2}}$. Indicate which rules of differentiation are being applied.

3) Two curves are *orthogonal* if their tangent lines are perpendicular at each point of intersection. Determine the value of the number a such that the curves $xy = 1$ and $xy^3 = a^3$ are orthogonal.

4) Show that the equation $x^3 + e^x = 0$ has exactly one solution.

5) Compute the 2nd-degree Taylor polynomial for the function $f(x) = x \ln x$ centered around the point where f attains its minimum.

6) Find a function F such that $F'(x) = x^3$ and the line $x+y = 0$ is tangent to the graph of F.

Instructions

- **write your name and student number on the top of each sheet of writing paper!**
- use the writing (lined) and scratch (blank) paper provided, raise your hand if you need more paper
- start each question on a new page
- use a pen with black or blue ink
- do not use any kind of correcting fluid or tape
- any rough work should be crossed through neatly so it can be seen
- this exam is open-book, you may use the textbook or the lecture notes
- you may view your textbook or notes on your devices (tablet/laptop/etc), but they have to be in airplane mode!
- you are allowed to use a simple pocket calculator
- programmable/graphing calculators are not allowed
- your work should be clearly and logically structured
- **explain your reasoning using words**
- show all your calculations, an answer without any computation will not be rewarded
- each problem is worth 15 points
- upon completion[1] submit your worksheets at the front desk

[1] At the end of the exam or after you finished, whichever is sooner.